Physics-Based Modeling of Electromagnetic Parasitic Effects in Interconnects

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Motivation

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The "Wiring Crisis"

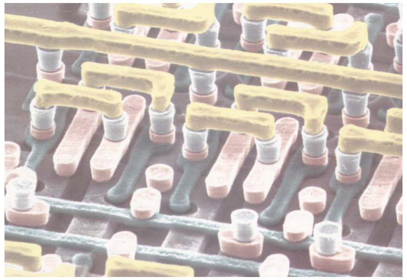
- Complex topologies and conductor shapes in multilevel interconnect structures
- Very short signal rise times (some ps) and ever-increasing clock frequency (towards 10 GHz)

⇓

Transient 3D-behavior of interconnects is becoming a limiting factor (more than 50% of signal delay results from interconnect wires)

⇓

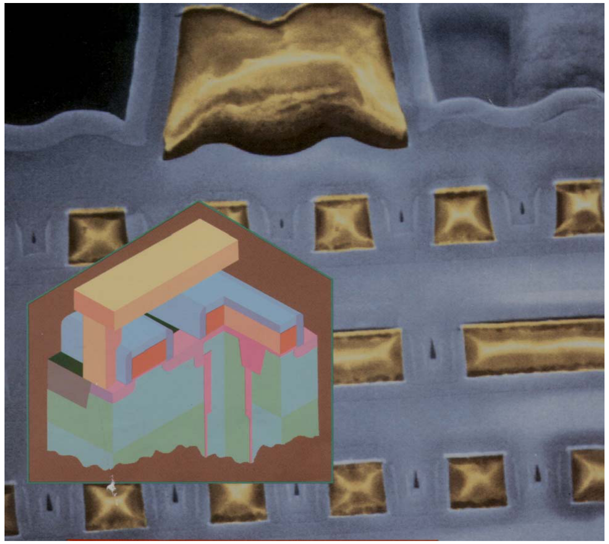
Predictive simulation of circuit behavior based on physical macromodels of interconnects is becoming indispensable for circuit design!



Interconnect Wires in SRAM Cell
(IBM/CMOS 5X process)

⟹ "virtual prototyping"

Multiple Layer Metallization for SRAM-Technology (IBM/CMOS 5X Process)



IBM J. Res. & Dev. Vol. 39(1/2)1995

Motivation (Cont'd)

Problems to be analyzed:

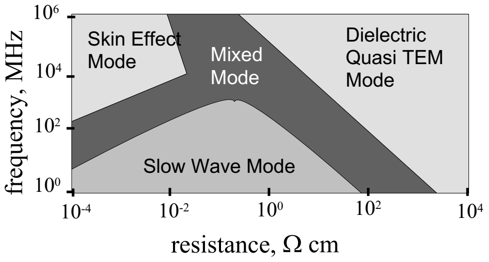
- Parasitic electromagnetic effects in signal propagation (delay, dispersive distortion of wave- and pulse-form, reflection and scattering, cross-talk between neighboring wires)
- Optimization of interconnect topology and topography

Methodological approach:

- Full coupling of dynamic (transient) behavior of devices and interconnects
- Macromodel of interconnects derived from distributed transient fields ("generalized impedance operator")

⇓

Physically-based full transient predictive circuit simulation



Map of signal propagation modes in integrated circuits

(adapted from: Hasegawa et al, IEEE Trans. on MTT, vol. 19, No. 11, 1971)

Problem Definition and Basic Approach

Interconnect Parasitics

can strongly affect the dynamic operation of an integrated circuit:

- ∗ influence on timing (delay times)
- ∗ cross-talk between neighboring interconnects
- ∗ inertia of electromagnetic field in surrounding matter (“slow wave mode”)
- ∗ damping effects in dielectric and conducting matter (⇒ dispersive distortion of waveforms and signals)
- ∗ wave / signal reflection and scattering on material discontinuities

⇒ analyze, understand and minimize electromagnetic parasitics

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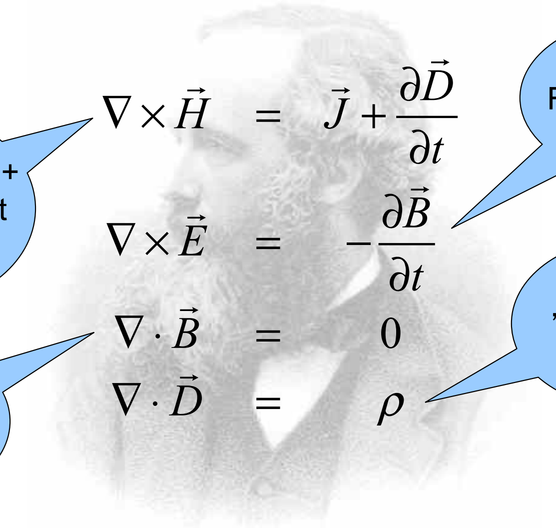
Physical Basis: Maxwell’s Equations



$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho$$

Ampere‘s law + Displacement current

Faraday‘s law

No Magnetic Charge

„Coulomb‘s law“

Maxwell's Equations in the QSA

Maxwell equations:

$$\nabla \times \vec{H} = \vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \cancel{\rho}$$

Constitutive Equations:

$$\vec{B} = \mu \vec{H}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{J} = \sigma \vec{E}$$

Initial conditions:

$$\vec{B}(t_0) = \vec{B}_0 \text{ in } \Omega_n \text{ and } \Omega_c$$

Boundary conditions:

$$\vec{E} \times \vec{n} = 0 \quad \text{on} \quad \Gamma_E$$

$$\vec{H} \times \vec{n} = 0 \quad \text{on} \quad \Gamma_H$$

Interface conditions:

$$\vec{H}_c \times \vec{n}_c + \vec{H}_n \times \vec{n}_n = 0 \quad \text{on} \quad \Gamma_{nc}$$

$$\vec{B}_c \cdot \vec{n}_c + \vec{B}_n \cdot \vec{n}_n = 0 \quad \text{on} \quad \Gamma_{nc}$$

Continuous Field Description of Inductive Parasitics

Maxwell's eqs.:

$$div\,\vec{D} = \rho = 0$$

$$div\,\vec{B} = 0$$

$$curl\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$curl\,\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

material relations:

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{j} = \sigma \vec{E}$$

potentials:

$$\vec{B} = curl\,\vec{A}$$

$$\vec{E} = -grad\,\varphi - \frac{\partial \vec{A}}{\partial t}$$

gauge:

$$div\,\vec{A} = 0$$

$$curl\,\tfrac{1}{\mu}\,curl\,\vec{A} + \sigma\dot{\vec{A}} + \varepsilon\,\ddot{\vec{A}} = -\sigma(grad\,\varphi + \tfrac{\varepsilon}{\sigma}\,grad\,\dot{\varphi})$$

$$\vec{j} = \sigma\vec{E} = -\sigma\,grad\,\varphi - \sigma\dot{\vec{A}}$$

decoupling of externally biased and induced current

potential flow eddy current

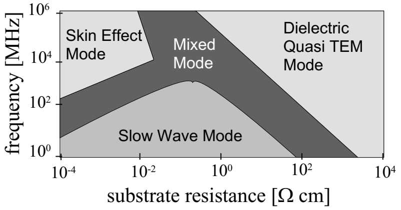
Field Diffusion Approximation

$$curl\,\tfrac{1}{\mu}\,curl\vec{A} + \sigma\dot{\vec{A}} + \varepsilon\,\ddot{\vec{A}} = -\sigma(grad\,\varphi + \tfrac{\varepsilon}{\sigma}\,grad\,\dot{\varphi})$$

$$|\,\tfrac{\varepsilon}{\sigma}\,grad\,\dot{\varphi}\,| \ll |\,grad\,\varphi\,|$$

$$|\,\tfrac{\varepsilon}{\sigma}\,\ddot{\vec{A}}\,| \ll |\,\dot{\vec{A}}\,|$$

- dielectric relaxation time $\frac{\varepsilon}{\sigma} \ll \tau_{switch}$
- no transmission of waves



$$\frac{\partial\vec{A}}{\partial t} - \frac{1}{\mu\sigma}\Delta\vec{A} = -\nabla\varphi$$

with $\nabla(\sigma\nabla\varphi) = 0$

$$\vec{j} = \vec{j}_{pot} + \vec{j}_{eddy}\,;$$

$$\vec{j}_{pot} = -\sigma\nabla\varphi\,; \qquad \vec{j}_{eddy} = -\sigma\dot{\vec{A}}$$

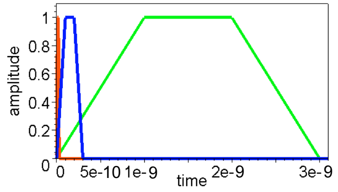
($\Rightarrow$ decoupling of effects)

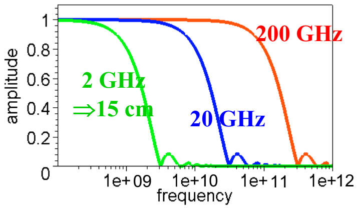
Quasistationary Approximation

Displacement current can be neglected

$$\frac{\partial\vec{D}}{\partial t} \ll \vec{J}_{eddy} \Rightarrow \frac{\partial\left(\varepsilon\vec{E}\right)}{\partial t} \ll \sigma\vec{E} \Rightarrow \frac{\varepsilon}{\sigma} \ll \tau_{switch}$$

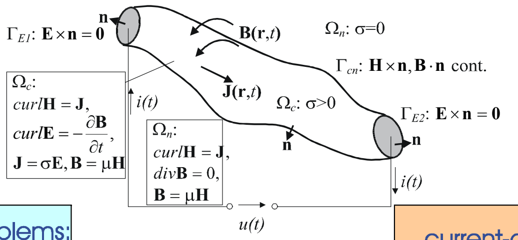
material	dielectric relaxation time
copper	1.5e-19 s
aluminum	2.5e-19 s
low doped Si (3 S/m)	3.5e-11 s
glass (1e-14 to 1e-12 S/m)	4.5 ... 4500 s





Typical geometrical length << shortest wavelength encountered

Potential Formulations for “Skin Effect” Problems



voltage-driven problems:

$$\mathbf{B}=curl\mathbf{A} \quad \text{in} \quad \Omega_c \text{ and } \Omega_n$$

$$\mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}-grad\frac{\partial v}{\partial t} \quad \text{in} \quad \Omega_c$$

current-driven problems:

$$\left.\begin{array}{l}\mathbf{J}=curl\left(\mathbf{T}_0+\mathbf{T}\right)\\ \mathbf{H}=\mathbf{T}_0+\mathbf{T}-grad\Phi\end{array}\right. \quad \text{in} \quad \Omega_c$$

$$\mathbf{H}=\mathbf{T}_0-grad\Phi \quad \text{in} \quad \Omega_n$$

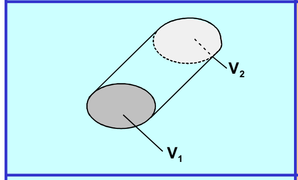
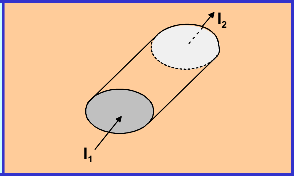
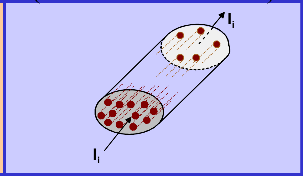
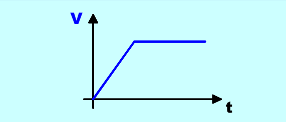
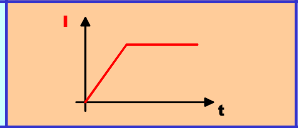
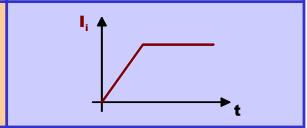
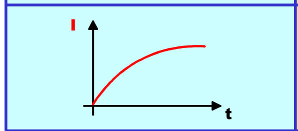
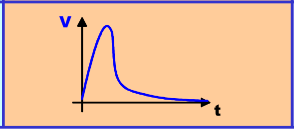
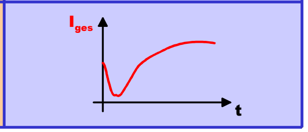
modified vector potential:

$$B=\nabla\times\vec{A}^* \text{ and } \vec{E}=-\frac{\partial\vec{A}^*}{\partial t}$$

$$\frac{\partial\vec{A}^*}{\partial t}-\frac{1}{\mu\sigma}\Delta\vec{A}^*=-\frac{1}{\sigma}\vec{j}_{pot}$$

with weak gauge $\nabla\cdot\left(\sigma\,\vec{A}^*\right)=0$

The Three Basic Regimes

Voltage-driven solid conductor	Current-driven solid conductor	Thin highly conducting threads sunk in a bulk (no stranded conductor)
		
		
		

A,V-A-Method

Basic equations:

$$\nabla\times\left(\frac{1}{\mu}\nabla\times\vec{A}\right)+\sigma\frac{\partial\vec{A}}{\partial t}+\sigma\, grad\frac{\partial v}{\partial t}=0$$

$$-\nabla\left(\sigma\frac{\partial\vec{A}}{\partial t}+\sigma\, grad\frac{\partial v}{\partial t}\right)=0 \qquad \text{in } \Omega_c \text{ ,}$$

$$\nabla\times\left(\frac{1}{\mu}\nabla\times\vec{A}\right)=0 \qquad \text{in } \Omega_{\mathrm{n}}$$

Boundary conditions:

$$\vec{A}\times\vec{n}=0\;,\;\; v(t)=\int_0^t\varphi(\tau)\,d\tau \qquad \text{on } \Gamma_{\mathrm{E1}}$$

$$\vec{A}\times\vec{n}=0\;,\;\; v(t)=0 \qquad \text{on } \Gamma_{\mathrm{E2}}$$

$$\vec{A}\times\vec{n}=0 \;\text{ or }\; \frac{1}{\mu}\nabla\times\vec{A}\times\vec{n}=0 \qquad \text{on } \partial\left(\Omega_{\mathrm{c}}+\Omega_{\mathrm{n}}\right)$$

$$\vec{A}\times\vec{n} \;\text{ and }\; \frac{1}{\mu}\nabla\times\vec{A}\times\vec{n} \qquad \text{are continuous on } \Gamma_{\mathrm{cn}}$$

T-T$_o$-Φ-Method

Basic equations:

$$\nabla\times\left(\frac{1}{\sigma}\nabla\times\vec{T}\right)+\frac{\partial}{\partial t}\left(\mu\vec{T}\right)-\frac{\partial}{\partial t}\left(\mu\, grad\Phi\right)=-\nabla\times\left(\frac{1}{\sigma}\nabla\times\vec{T}_0\right)-\frac{\partial}{\partial t}\left(\mu\vec{T}_0\right)$$

$$\frac{\partial}{\partial t}\nabla\cdot\left(\mu\vec{T}_0+\mu\vec{T}-\mu\, grad\Phi\right)=0 \qquad \text{in } \Omega_c \text{ ,}$$

$$\frac{\partial}{\partial t}\nabla\cdot\left(\mu\vec{T}_0-\mu\, grad\Phi\right)=0 \qquad \text{in } \Omega_{\mathrm{n}}$$

Boundary conditions:

$$\frac{1}{\sigma}\nabla\times\vec{T}\times\vec{n}=0\;,\; \mu\left(\vec{\mathrm{T}}_0+\vec{\mathrm{T}}\text{-grad}\Phi\right)\cdot\vec{n}=0 \qquad \text{on } \Gamma_{\mathrm{E1}},\Gamma_{\mathrm{E2}}$$

$$\vec{T}\times\vec{n}=0 \qquad \text{on } \Gamma_{\mathrm{cn}}$$

$$\Phi=0 \;\text{ or }\; \mu\left(\vec{\mathrm{T}}_0\text{-grad}\Phi\right)\cdot\vec{n}=0 \qquad \text{on } \Gamma_{\mathrm{c}}\text{ , }\Gamma_{\mathrm{n}}$$

$$\Phi \;\text{ and }\; \mu\left(\vec{\mathrm{T}}_0\text{-grad}\Phi\right)\cdot\vec{n} \qquad \text{are continuous on } \Gamma_{\mathrm{cn}}$$

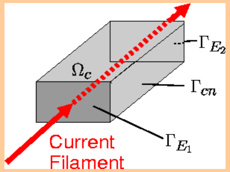
T-T$_o$-Φ -Method (cont'd)

How to construct $\vec{T}_0$

$$\nabla\times\left(\frac{1}{\sigma}\nabla\times\vec{T}_0\right)=0 \quad \text{in } \Omega_c$$

$$\vec{T}_0\times\vec{n}=\vec{H}_S\times\vec{n} \qquad \text{on } \Gamma_{cn}\ (\hat{=}\,\Gamma_{outer})$$

$$\frac{1}{\sigma}\nabla\times\vec{T}_0\times\vec{n}=0 \qquad \text{on } \Gamma_{E1} \text{ and } \Gamma_{E2}$$

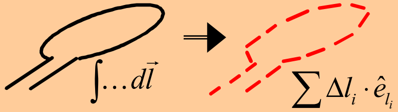


How to get

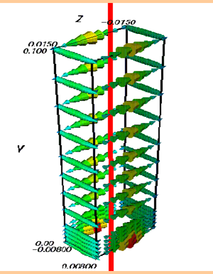
$\vec{H}_S\times\vec{n}$ on Γ_{cn}

Biot-Savart-Field:

$$d\vec{B}=\frac{\mu_0 I}{4\pi}\cdot\frac{d\vec{l}\times\hat{r}}{r^2}$$

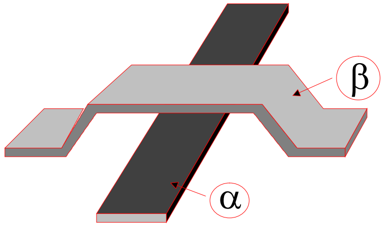


$\int\ldots d\vec{l}$ ⇒ $\sum\Delta l_i\cdot\hat{e}_{l_i}$



Transient Inductance Effects on Circuit Level

Description of Inductive Parasitics on Circuit Level



Quasistatic approximation:

$$U_\alpha^{(ind)}=\sum_\beta L_{\alpha\beta}\frac{dI_\beta}{dt}$$

$$L_{\alpha\beta}=\frac{\mu}{4\pi}\int_{\Omega_\alpha}\int_{\Omega_\beta}\frac{\vec{j}_\alpha(\vec{r})\cdot\vec{j}_\beta(\vec{s})}{|\vec{r}-\vec{s}|}d^3r\,d^3s$$

with $\vec{j}_\alpha(\vec{r})=$ current basis functions

- Conventional approach in most interconnect analysis tools.

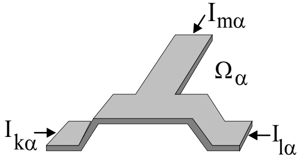
Problem: QSA does not allow for eigendynamics of interconnects embedded in surrounding materials (induced current distributions, damping, ...)

Response Function on Circuit Level (1)

1. Separation of space and time variables:

Input:

Terminal currents $I_{k\alpha}(t)$
at device contact $C_{k\alpha}$
on interconnect part Ω_α



⇒ quasistationary current flow built up from basis functions

$$\boxed{\vec{j}_{qs}(\vec{r},t)=\sum_{k,\alpha}\vec{j}_{k\alpha}(\vec{r})I_{k\alpha}(t)}$$

where $\vec{j}_{k\alpha}=-\sigma_\alpha\nabla\phi_{k\alpha}$ is calculated from $div(\sigma_\alpha\nabla\phi_{k\alpha})=0$

satisfying boundary conditions $\int_{C_{j\alpha}}\vec{j}_{k\alpha}d\vec{a}=\delta_{kj}-\frac{1-\delta_{kj}}{N-1}$

Response Function on Circuit Level (2)

2. Induced field and currents:

$\vec{A}(\vec{r},t) = \sum_{k,\alpha} \int \vec{A}_{k\alpha}(\vec{r},t-\tau) I_{k\alpha}(\tau) d\tau$ using

basis solutions of "diffusion equation" $\frac{\partial \vec{A}_{k\alpha}}{\partial t} - \frac{1}{\mu\sigma}\Delta \vec{A}_{k\alpha} = \frac{1}{\sigma}\vec{j}_{k\alpha}(\vec{r})\delta(t)$

$$\Rightarrow \quad \boxed{\vec{j}(\vec{r},t) = \sum_{k,\alpha} \vec{j}_{k\alpha}(\vec{r}) I_{k\alpha}(t) - \sigma_\alpha \int \vec{A}_{k\alpha}(\vec{r},t-\tau)\dot{I}_{k\alpha}(\tau)d\tau}$$

3. Generalized inductance matrix:

Magnetic field energy $\int \vec{A}\cdot\vec{j} d^3r$ includes inductively stored contribution W_{ind}

with $\frac{dW_{ind}}{dt} = \sum_{k,\alpha}\sum_{l,\beta}\int_{-\infty}^{t} I_{k\alpha}(t) L_{k\alpha,l\beta}(t-\tau)\dot{I}_{l\beta}(\tau)d\tau$

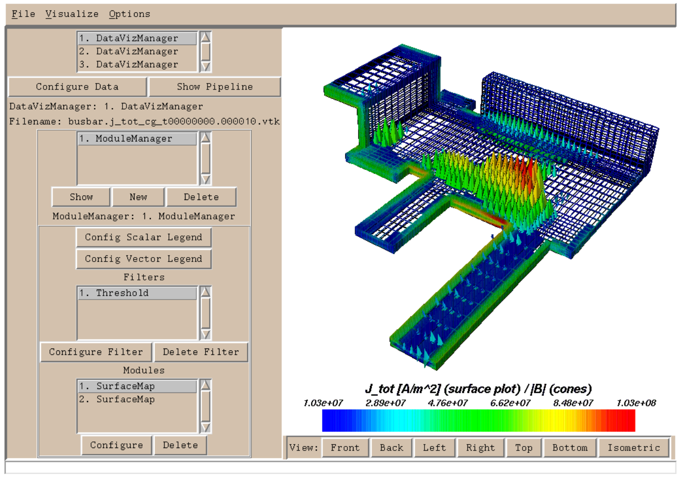
$$\text{where} \quad \boxed{L_{k\alpha,l\beta}(\tau) = \int_{\Omega_\alpha} \vec{j}_{k\alpha}(\vec{r})\vec{A}_{l\beta}(\vec{r},\tau)d^3r = \frac{1}{\sigma_\alpha}\langle \vec{j}_{k\alpha}|e^{-\mathcal{D}\tau}\vec{j}_{l\beta}\rangle}$$

Implementation in
New Numerical Software Package

The PDE-C++ Library Diffpack

- sophisticated **tool for developing numerical software** with main emphasis on **numerical solution** of **partial differential equations**
- collection of **C++** libraries with **classes,** functions, utility programs
- large collection of useful abstractions: vectors, matrices, general multi-index arrays, strings, improved and simplified I/O, menu system, management of result files, coupling to visualization tools, FEM modules, FDM modules, ...
- Advanced add-on tool boxes: adaptivity, input-/output-filters, multilevel, mixed finite element method, parallelization, domain decomposition, ...

Postprocessing by MayaVi

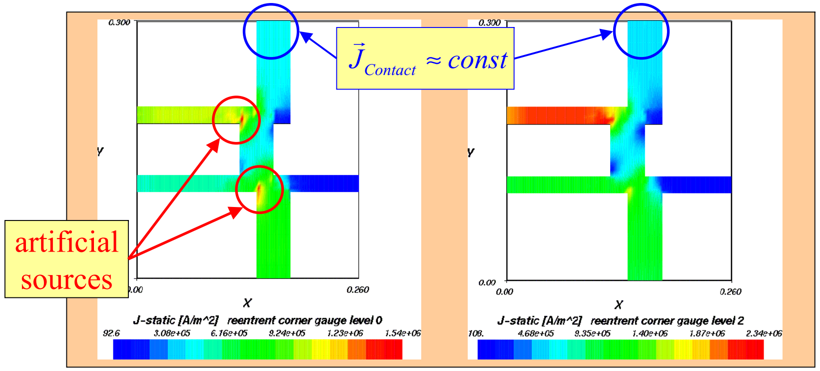


DataVizManager: 1. DataVizManager

Filename: busbar.j_tot_cg_t00000000.000010.vtk

ModuleManager: 1. ModuleManager

T_o-Problem & Node Elements



$\vec{J}_{Contact} \approx const$

artificial sources

	I(soll)	I(gl=0)	I(gl=2)
Contact 1	200 A	180 A	200 A
Contact 2	100 A	60 A	100 A
Contact 3	200 A	100 A	200 A
Contact 4	100 A	90 A	100 A
Iterations (CG)		112	328

~~Node elements & current driven problems~~

The Degrees of Freedom

Degree of freedom: The degrees of freedom are the moments of the tangential component along each edge

$$\int_{\hat{e}} \hat{u} \cdot \hat{\tau} \, ds$$

for each edge $\hat{e}$ of $\hat{K}$ with unit tangent $\hat{\tau}$.

Simplest edge element: **Nedelec element with constant tangential and linear normal component (CTLN)**

- $\vec{N}_i$ has constant tangential component on edge i
- no tangential component on all other edges

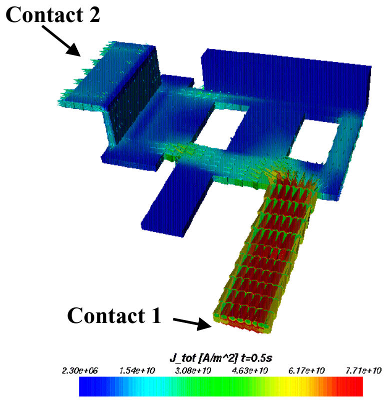
Mixed FEM & Edge Elements

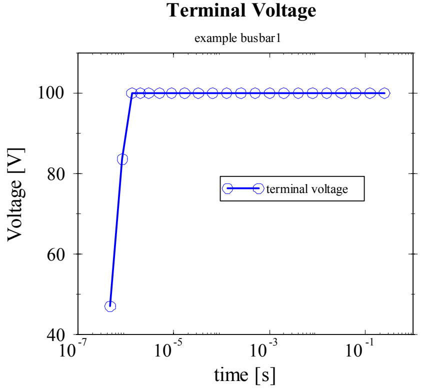
- **Mixed finite element method** allows for **easy extension** of library by new kinds of **vector-based elements**
  - Three vector components ⇒ three different elements on local coordinate system level.
  - Only one result field needed to represent the degree of freedoms, the path integrals of the vector field along the edges.
  - special mapping functionality responsible for: unique edge direction, right path integration, curl-conforming mapping.
- Combination of vector and scalar degree of freedom possible

⇒ „double“ mixed finite element method

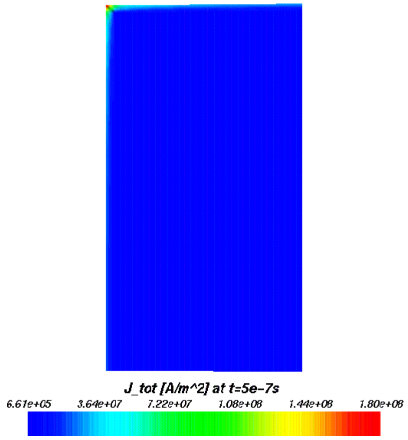
Elementary Examples

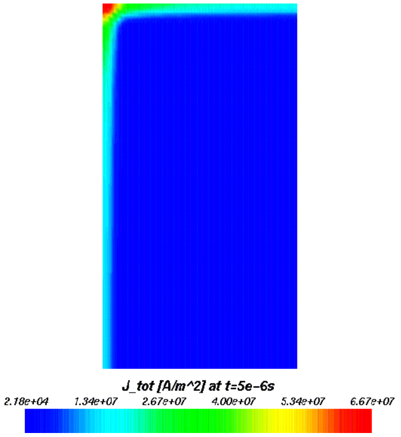
Example A,V-A



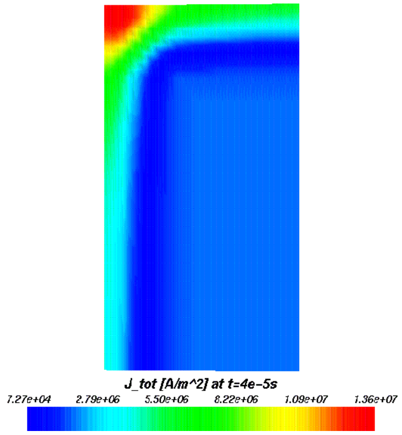


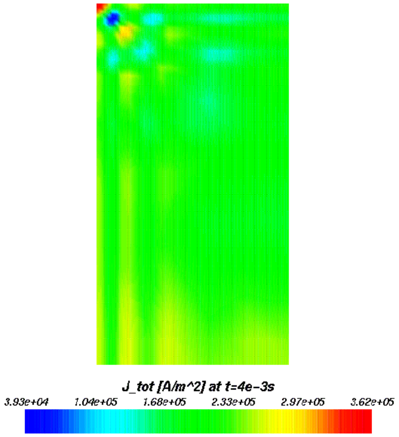
Scalar Cut of Current Density





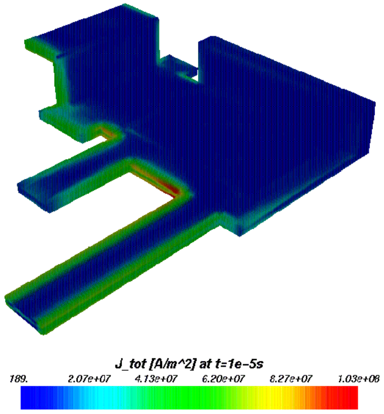
Scalar Cut of Current Density



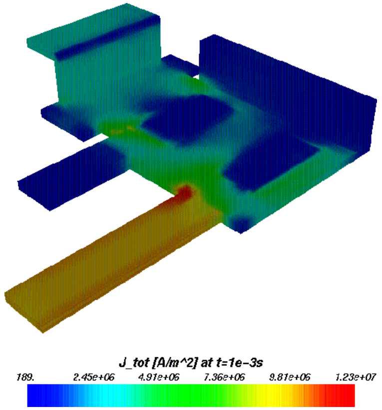


Current-Driven Busbar

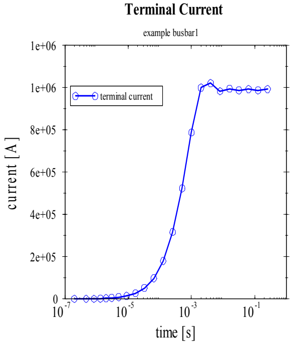
Skin Effect State

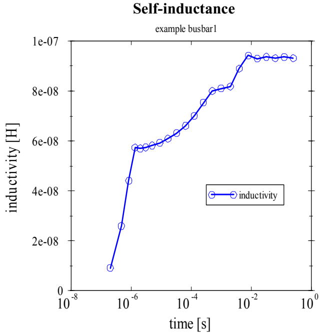


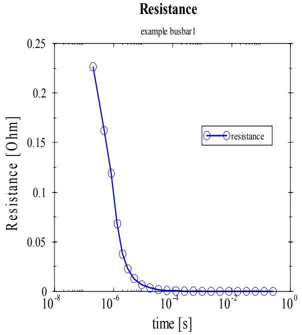
Static State



Example A,V-A


Terminal Current

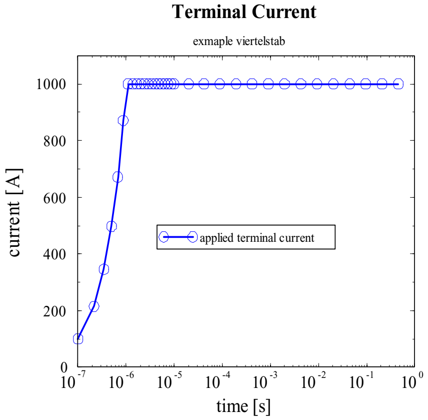

Self-inductance


Resistance

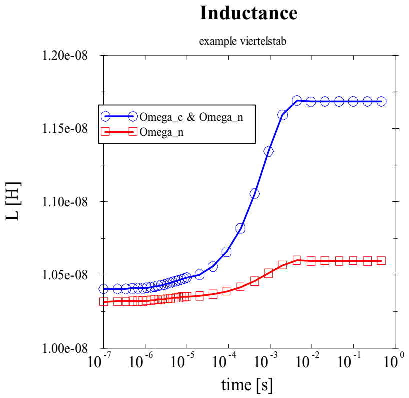
Example T-T$_o$-Φ

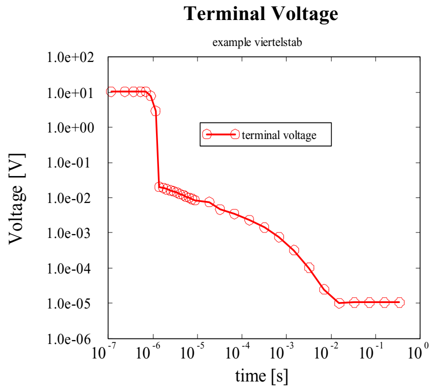
Current-driven problem solved using current potential method:

Apply current ramp at contact, find time-dependent inductance and terminal voltage as response.

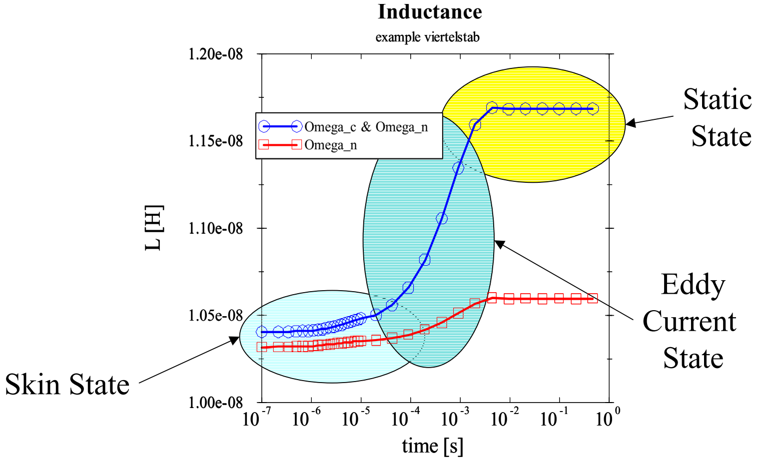

Terminal Current

Example T-T$_o$-Φ


Inductance


Terminal Voltage

Three Different States


Inductance

Résumé of Numerical Tests

- **Potential formulations** are well suited to **voltage- and current-driven problems** (“2 1/2“ different formulations possible).
- FE-simulations allow **identification** (and optimization) of various **distributed parasitic effects.**
- Diffpack proves to be a powerful platform for building a simulator framework for “real world“ industrial problems.
- Further enhancements of simulator easily possible due to modular structure of the libraries.

Conclusions

* Coping with the "wiring crisis“ necessitates physically-based macromodeling for predictive transient interconnect simulation in RF circuits.
* Accurate 3D-analysis of parasitic electromagnetic effects has to be based on "tailored“ distributed transient field model (= problem-oriented reduced version of Maxwell’s equations = "field diffusion approximation“, FDA).
* Proper gauge of electromagnetic 4-potential decisive for numerically robust treatment of "real world problems“ (current-driven $\neq$ voltage-driven).
* New concept of "impedance operator“ (= generalized time-dependent inductance matrix) seems adequate for proper transient interconnect analysis on circuit level.
* Implementation of method in new numerical simulation package based on PDE-C++ DIFFPACK^{TM} library and edge element discretization.